M463 Homework 7

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(2.2) #6 To estimate the percent of district voters who oppose a certain ballot measure, a survey organization takes a random sample of 200 voters from a district. If 45% of the voters in the district oppose the measure, estimate the chance that:

(Assume that all voters in the district are equally likely to be in the sample, independent of each other.)

a) exactly 90 voters in the sample oppose the measure.

Solution: Let X = the number of people in the sample that oppose the measure. Note that X has a binomial distribution with $n = 200$ and $p = 0.45$ where p is the probability that a selected voter opposes the measure. In this setting we have:

$$P(\text{90 successes in 200 trials}) = P(X = 90) = \binom{200}{90} 0.45^{90} 0.55^{110} = \boxed{0.056631399}$$

We could have instead approximated this value using the normal distribution with the following parameters: $\mu = np = 200 \cdot 0.45 = 90$ and $\sigma = \sqrt{npq} = 7.03562364$. Hence,

$$\begin{aligned} P(X = 90) = P(89.5 \leq X \leq 90.5) &= P(X \leq 90.5) - P(X \leq 89.5) \\ &= P\left(\frac{X-\mu}{\sigma} \leq \frac{90.5 - 90}{7.03562364}\right) - P\left(\frac{X-\mu}{\sigma} \leq \frac{80.5 - 90}{7.03562364}\right) \\ &\approx P(Z \leq 0.071066905) - P(Z \leq -0.071066905) \\ &= 2P(Z \leq 0.071066905) - 1 \\ &= \boxed{0.056655492} \end{aligned}$$

Note that the error of the estimate is very small: $\epsilon = 0.056655492 - 0.056631399 = 0.000024093$

b) more than half the voters in the sample oppose the measure.

Solution:

$$P(\text{more than 100 successes in 200 trials}) = P(X > 100) = \sum_{i=101}^{200} \binom{200}{i} 0.45^{i} 0.55^{200-i} = \boxed{0.06807525}$$

Again, we could have instead approximated this value using the normal distribution with the same parameters as before. Hence,

$$\begin{aligned} P(X > 100) = 1 - P(X \leq 100) &= 1 - P\left(\frac{X-\mu}{\sigma} \leq \frac{100.5 - 90}{7.03562364}\right) \\ &\approx P(Z \leq 1.492405014) \\ &= \boxed{0.067796501} \end{aligned}$$

In this case the error of the estimate is also very small: $\epsilon = 0.06807525 - 0.067796501 = 0.000278749$